

# Quantifying entanglement in macroscopic systems

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**Traditionally, entanglement was considered to be a quirk of microscopic objects that defied a common-sense explanation. Now, however, entanglement is recognized to be ubiquitous and robust. With the realization that entanglement can occur in macroscopic systems — and with the development of experiments aimed at exploiting this fact — new tools are required to define and quantify entanglement beyond the original microscopic framework.**

In the past decade, there has been an explosion of interest in entanglement in macroscopic (many body) physical systems<sup>1</sup>. The transformation in how entanglement is perceived has been remarkable. In less than a century, researchers have moved from distrusting entanglement because of its ‘spooky action at a distance’ to starting to regard it as an essential property of the macroscopic world.

There are three basic motivations for studying entanglement in the macroscopic world. The first motivation is fundamental. Researchers want to know whether large objects can support entanglement. The conventional wisdom is that a system that consists of a large number of subsystems (for example,  $10^{26}$  of them, similar to the number of atoms in a living room) immersed in an environment at a high temperature (room temperature, for example) ought to behave fully classically. Studying macroscopic entanglement is thus a way of probing the quantum-to-classical transition.

The second motivation is physical and relates to the different phases of matter. Traditionally, the idea of an order parameter is used to quantify phase transitions. For example, a non-magnetic system (in the ‘disordered phase’) can be magnetized (or become ordered) in certain conditions, and this transition is indicated by an abrupt change in the order parameter of the system. In this case, the magnetization itself is a relevant order parameter, but the interesting question is whether entanglement is a useful order parameter for other phase transitions<sup>2,3</sup>.

The third motivation comes from technology. If the power of entanglement is to be harnessed through quantum computing, then entangled

systems of increasingly large sizes need to be handled, which is itself a challenge.

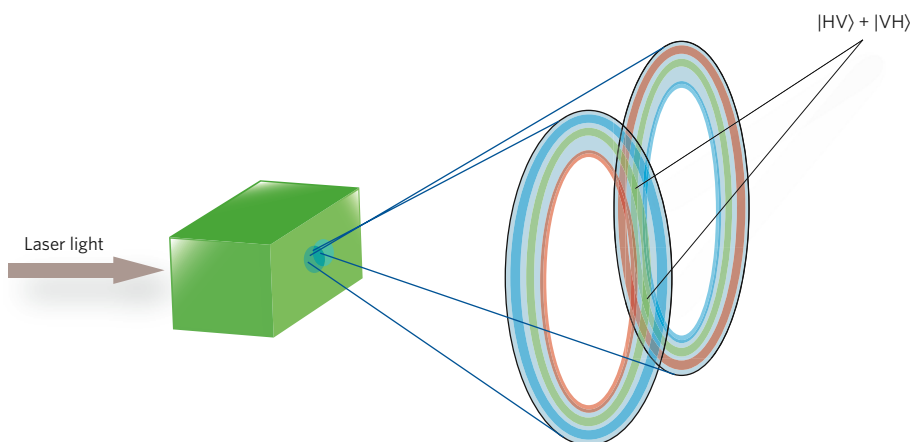
It is clear that the modern perspective on entanglement differs greatly from the initial ideas about its seemingly paradoxical nature. Researchers are now realizing how general and robust entanglement is. Larger and larger entangled systems are being manipulated coherently in different physical implementations. And it is not as surprising as it once was to find that entanglement contributes to some phenomena.

Not all of the mystery has vanished, however. As is common in scientific research, answering one question generates many new ones, in this case related to the type of entanglement that is useful for studies motivated by each of the three reasons above. These questions bring researchers closer to the heart of the current understanding of entanglement.

Here I first examine what entanglement is and how it is quantified in physical systems. Different classes of entanglement are then discussed, and I conclude by considering the possibilities of achieving and exploiting large-scale entanglement in the laboratory.

## What is entanglement?

The first chapter of almost any elementary quantum-mechanics textbook usually states that quantum behaviour is not relevant for systems with a physical size much larger than their de Broglie wavelength. The de Broglie wavelength, which can intuitively be thought of as the quantum extent of the system, scales inversely as (the square root of) mass



**Figure 1 | A way of generating entangled photons by using down conversion.** The input laser light is shone onto a nonlinear crystal (green box). The nonlinearity of the crystal means that there is a non-zero probability that two photons will be emitted from the crystal. The cones represent the regions where each of the two photons is emitted. Owing to energy conservation, the frequencies of the photons need to add up to the original frequency. Their momenta also must cancel in the perpendicular direction and add up to the original momentum in the forward direction. One of the photons is horizontally polarized (H), and one is vertically polarized (V). However, in the regions where the two cones overlap, the state of the photons will be  $|HV\rangle + |VH\rangle$ . It is around these points that entangled photons are generated.

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times temperature. From this, it can be concluded that massive and hot systems — which could almost be considered as synonymous with macroscopic systems — should not behave quantum mechanically.

As I show in the next section, however, de-Broglie-type arguments are too simplistic. First, entanglement can be found in macroscopic systems<sup>4</sup> (including at high temperatures<sup>5</sup>). And, second, entanglement turns out to be crucial for explaining the behaviour of large systems<sup>6</sup>. For example, the low values of magnetic susceptibilities in some magnetic systems can be explained only by using entangled states of those systems.

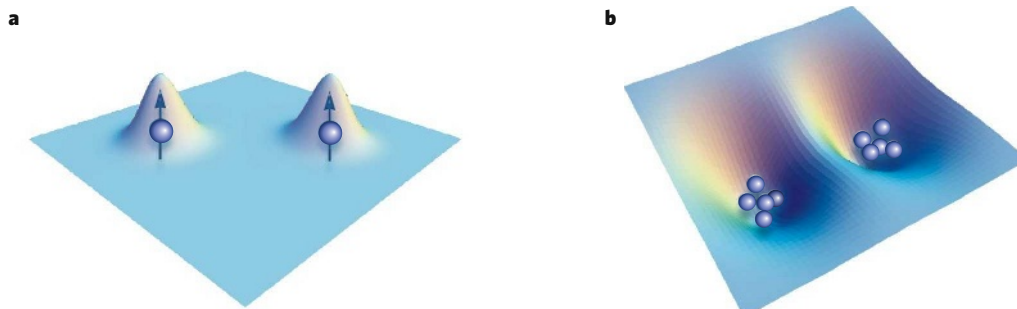
Now, what exactly is entanglement? After all is said and done, it takes (at least) two to tangle<sup>7</sup>, although these two need not be particles. To study entanglement, two or more subsystems need to be identified, together with the appropriate degrees of freedom that might be entangled. The subsystems are technically known as modes, and the possibly entangled degrees of freedom are called observables. Most formally, entanglement is the degree of correlation between observables pertaining to different modes that exceeds any correlation allowed by the laws of classical physics.

I now describe several examples of entangled systems. Two photons that have been generated by, for example, parametric down conversion<sup>8</sup> are in the overall polarized state  $|HV\rangle + |VH\rangle$  (where H is horizontal polarization and V is vertical polarization) and are entangled as far as their polarization is concerned (Fig. 1). A photon is an excitation of the electromagnetic field, and its polarization denotes the direction of the electric field. Each of the two entangled photons represents a subsystem, and the relevant observables are the polarizations in different directions. (Two electrons could also be entangled in terms of their spin value in an analogous way.)

When two subsystems in pure states become entangled, the overall state can no longer be written as a product of the individual states (for example,  $|HV\rangle$ ). A pure state means that the information about how the state was prepared is complete. A state is called mixed if some knowledge is lacking about the details of system preparation. For example, if the apparatus prepares either the ground state  $|0\rangle$  or the first excited state  $|1\rangle$  in a random manner, with respective probabilities  $p$  and  $1-p$ , then the overall state will need to be described as the mixture  $p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$ . In this case, the probabilities need to be used to describe the state because of the lack of knowledge. Consequently, quantifying entanglement for mixed states is complex.

Systems can also be entangled in terms of their external degrees of freedom (such as in spatial parameters). For example, two particles could have their positions and momenta entangled. This was the original meaning of entanglement, as defined by Albert Einstein, Boris Podolsky and Nathan Rosen<sup>9</sup>.

When the subsystems have been identified, states are referred to as entangled when they are not of the disentangled (or separable) form<sup>10</sup>:  $\rho_{\text{sep}} = \sum_i p_i \rho_1^i \otimes \rho_2^i \otimes \dots \otimes \rho_n^i$ , where  $\sum_i p_i = 1$  is a probability distribution and  $\rho_1^i, \rho_2^i, \dots, \rho_n^i$  are the states (generally mixed) of subsystem 1, 2, ...,  $n$ ,



**Figure 2 | Separable states.** Two examples of disentangled systems are shown. **a**, Two electrons are shown confined to two spatial regions and with their internal spins pointing up. In this case, their spin states are both in the same upwards direction. Because electrons are fermions, the overall state of this system must be antisymmetrical. The internal state is symmetrical (because the electrons are pointing up), and so their spatial wavefunctions must be antisymmetrized,  $|\Psi_1\Psi_2\rangle - |\Psi_2\Psi_1\rangle$ . The spatial part of the electronic state, therefore, seems to be entangled — but this only seems to be the case. The

respectively. On the one hand, two subsystems described by the density matrix  $\rho_{12} = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$  are one such example of a separable state. The state of three subsystems,  $|000\rangle + |111\rangle$ , on the other hand, can easily be confirmed to be not separable and therefore (by definition) entangled.

This simple mathematical definition hides a great deal of physical subtlety. For example, Bose–Einstein condensates are created when all particles in a system go into the same ground state. It seems that the overall state is just the product of the individual particle states and is therefore (by definition) disentangled. However, in this case, entanglement lies in the correlations between particle numbers in different spatial modes. Systems can also seem to be entangled but, on closer inspection, are not (Fig. 2).

### Witnesses and measures of entanglement

In this section, I present two surprising results from recent studies of many-body entanglement: first, entanglement can be witnessed by macroscopic observables<sup>11,12</sup> (see the subsection ‘Witnessing entanglement’); and second, entanglement can persist in the thermodynamic limit at arbitrarily high temperatures<sup>13</sup>. The first statement is surprising because observables represent averages over all subsystems, so it is expected that entanglement disappears as a result of this averaging. The effect of temperature is similar. Increasing the temperature means that an increasing amount of noise is added to the entanglement, so the second finding — that entanglement can persist at high temperatures — is also surprising.

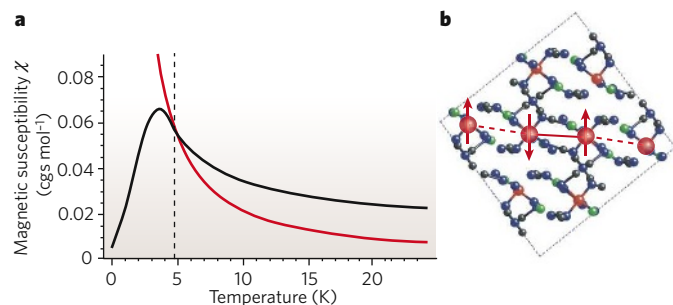
Before these findings are described in more detail, a simple observation can be made. The entanglement of two subsystems in a pure state is very easy to quantify. This is because the more entangled the state, the more mixed the subset of the system. This property of quantum states — namely that although exact information about the overall state is available, information about parts of the system can be incomplete — was first emphasized by Erwin Schrödinger<sup>14</sup>, in the famous paper in which he described the ‘Schrödinger’s cat’ thought experiment.

This logic fails for mixed states, however. For example, an equal mixture of  $|00\rangle$  and  $|11\rangle$  also results in maximally mixed states for each quantum bit (qubit), but the overall state is not entangled. It also fails for quantifying quantum correlations between more than two components. In fact, in this last case, it is even difficult to determine whether a state of many subsystems is entangled in the first place. This leads on to the concept of witnessing entanglement.

### Witnessing entanglement

Entanglement witnesses<sup>15</sup> are observables whose expectation value can indicate something about the entanglement in a given state. Suppose that there is an observable  $W$ , which has the property that for all disentangled states, the average value is bounded by some number  $b$ ,

electrons in question are fully distinguishable (because they are far apart), so any experiment on one of them is not correlated to any experiment on the other one. Therefore, these electrons cannot be entangled. **b**, A double-well potential, with each well containing five particles, is shown. Experiments that trap atoms in this way are now routine. It is clear that there is no entanglement between the two wells, because each well contains a fixed, clearly defined number of particles, although there could be some entanglement within each well, depending on the exact circumstances.



**Figure 3 | Susceptibility as a macroscopic witness of entanglement.** **a**, The typical behaviour of magnetic susceptibility  $\chi$  versus temperature for a magnetic system is depicted (black). The behaviour of the entanglement witness is also depicted (red). Values of magnetic susceptibility below the red line are entangled, and the dashed line indicates the transitional point. **b**, One of the earliest experimental confirmations of entanglement<sup>6</sup> involved copper nitrate,  $\text{Cu}(\text{NO}_3)_2 \cdot 2.5\text{H}_2\text{O}$ , with entanglement existing at less than  $\sim 5$  K. A molecular image of copper nitrate is depicted, with copper in red, nitrogen in blue, oxygen in black and hydrogen in green. The one-dimensional chain (red), which consists of interacting copper atoms, is the physically relevant chain in terms of the magnetic properties of the compound and can be thought of as a collection of dimers (shown separated by dashed red lines).

$\langle W \rangle \leq b$ . Suppose, furthermore, that a researcher is given a physical state and experimentally shows that  $\langle W \rangle > b$ , then the only explanation is that the state is entangled.

Imagine two spins (a ‘dimer’) coupled through a Heisenberg interaction<sup>4</sup>:  $H = -J\sigma \cdot \sigma$ , where  $H$  is the hamiltonian,  $\sigma$  denotes a Pauli spin matrix and  $J$  is the strength of coupling. I now use the hamiltonian as an entanglement witness. It is easy to see that the average value of  $H$  with respect to disentangled (separable) states cannot exceed the value  $J$ :  $\langle H \rangle = |\text{Tr}(\rho_{\text{sep}} H)| = J|\langle \sigma \rangle \langle \sigma \rangle| \leq J$ . However, if the expected values are computed for the singlet state (which is the ground state of  $H$ ), then the following is obtained:  $|\text{Tr}(\rho_{\text{sin}} H)| = 3J$ , where  $\rho_{\text{sin}}$  is the density matrix of the singlet state. This value is clearly outside the range of separable states. The singlet is therefore entangled. This logic generalizes to more complex hamiltonians (with arbitrarily many particles), and it can be shown that observables other than energy (for example, magnetic susceptibility) can be good witnesses of entanglement<sup>1</sup> (Fig. 3). In fact, by using this method, ground states of antiferromagnets, as well as other interacting systems, can generally be shown to be entangled at low temperatures ( $k_B T \leq J$ , where  $k_B$  is the Boltzmann constant and  $T$  is temperature; this seems to be a universal temperature bound for the existence of entanglement<sup>16</sup>).

### Measuring entanglement

Measuring entanglement is complex, and there are many approaches<sup>17</sup>. Here I discuss two measures of entanglement: overall entanglement and connectivity. Further measures are described in ref. 17.

The first measure is overall entanglement, also known as the relative entropy of entanglement<sup>18</sup>, which is a measure of the difference between a given quantum state and any classically correlated state. It turns out that the best approximation to the Greenberger–Horne–Zeilinger (GHZ)<sup>19</sup> state,  $|000\rangle + |111\rangle$ , is a mixture of the form  $|000\rangle\langle 000| + |111\rangle\langle 111|$ . For W states<sup>20</sup> (by which I mean any symmetrical superposition of zeros and ones, such as  $|001\rangle + |010\rangle + |100\rangle$ ), the best classical approximation is a slightly more elaborate mixture<sup>5</sup>.

On the basis of overall entanglement, W states are more entangled than GHZ states. What is the most entangled state of  $N$  qubits according to the overall entanglement? The answer is that the maximum possible overall entanglement is  $N/2$ , and one such state that achieves this (by no means the only one) is a collection of dimers (that is, maximally entangled pairs of qubits). This is easy to understand when considering that each dimer has one unit of entanglement and that there are  $N/2$  dimers in total.

The second measure (originally termed disconnectivity) is referred to here as connectivity<sup>21</sup>. Measuring connectivity is designed to address the

question of how far correlations stretch. Take a GHZ state of  $N$  qubits,  $|000 \dots 0\rangle + |111 \dots 1\rangle$ . It is clear that the first two qubits are as correlated as the first and the third and, in fact, as the first and the last. Correlations of GHZ states therefore have a long range. GHZ states have a connectivity equal to  $N$ . The W state, by contrast, can be well described by nearest-neighbour correlations. The W state containing  $N/2$  zeros and  $N/2$  ones can be well approximated by the states  $|01\rangle + |10\rangle$  between nearest neighbours. Therefore, correlations do not stretch far, and the connectivity is only equal to 2.

The above considerations of how to quantify entanglement are general and apply to all discrete (spin) systems, as well as to continuous systems (such as harmonic chains<sup>22</sup> and quantum fields<sup>23</sup>), although continuous systems need to be treated with extra care because of their infinite dimensionality. Although the discussed witnesses and measures can be applied to mixed states, I now focus on pure states for simplicity.

### Different types of macroscopic entanglement

There are many types of entanglement. Here I discuss the four types that cover all three motivations mentioned earlier: GHZ, W, resonating valence bond (RVB)<sup>24</sup> and cluster<sup>25</sup>. GHZ states are typically used in testing the non-locality of quantum mechanics, because they have a high value of connectivity. W and RVB states naturally occur for a range of physical systems. For example, both Bose–Einstein condensates (such as superfluid and superconducting materials) and ferromagnets have W states as ground states<sup>5</sup>.

RVB states are built from singlet states between pairs of spins. It is clear that connectivity of RVBs is only 2, but the states themselves have a high overall degree of entanglement,  $N/2$  (ref. 26). It is intriguing that natural states have low connectivity but a high overall entanglement that scales as  $\log N$  or even  $N/2$ , whereas GHZ states, which do not occur naturally, have high connectivity of the order of  $N$  but a very low overall entanglement (Box 1).

Are there states that have both connectivity and overall entanglement that scale as the number of subsystems? The answer is, surprisingly, yes. Even more interestingly, these states, which are known as cluster states, are important for quantum computing<sup>25</sup>. Cluster states are highly entangled arrays of qubits, and this entanglement is used to carry out quantum computing through single qubit measurements. Entanglement drives the dynamics of these computers<sup>27</sup>, which is why high overall entanglement is needed. But the type of entanglement is also responsible for the implementation of various gates during the operation of these computers, which is why high connectivity is needed.

### Experimental considerations and beyond

There are many paths to preparing and experimenting with larger collections of entangled systems. As I have described, natural entanglement is not strong in general and is far from being maximal with respect to overall entanglement or connectivity. To create high overall entanglement and connectivity invariably involves a great deal of effort.

There are two basic approaches to generating large-scale entanglement: bottom up and top down. The first approach, the bottom-up approach, relies on gaining precise control of a single system first and then extending this control to two systems and scaling it up further. So far, ‘bottom-up experiments’ have obtained up to eight entangled ions in an ion trap (in a W state)<sup>28</sup> and six entangled photons<sup>29</sup>. During nuclear magnetic resonance spectroscopy, 13 nuclei can be ‘pseudo-entangled’<sup>30</sup>. Larger systems, however, are exceedingly difficult to control in this way.

The second approach is the top-down approach. As described earlier, many natural systems, with many degrees of freedom (1 million atoms, for example), can become entangled without the need for difficult manipulations (for example, the only requirement might be to decrease the temperature to less than 5 K, which is physically possible). Moreover, in many systems, certain types of entanglement are present in thermal equilibrium and even above room temperature, without the need for any manipulation.

**Box 1 | Comparison of four types of entangled qubit state**

Qubit state	Overall entanglement	Connectivity
GHZ	1	$N$
W	$\log N$	2
RVB	$N/2$	2
Cluster	$N/2$	$N$

The naturally occurring states, W and RVB, have a much smaller connectivity than the states used for testing non-locality (GHZ) and for carrying out universal computing (cluster). In contrast to connectivity, the overall entanglement shows a different scaling. The important point is that the overall entanglement and connectivity capture markedly different aspects of the 'quantumness' of macroscopic states. Both of these measures can be thought of in terms of fragility of the entangled state, but they describe different types of fragility. The connectivity is related to the fragility of the state under dephasing: that is, the loss of phases between various components in the superposition. The overall entanglement, by contrast, is related to the fragility of the state under the full removal of qubits from the state. For example, if one qubit is removed from the GHZ state, then the remaining qubits automatically become disentangled, which is why the overall entanglement of the GHZ state is equal to 1. If each qubit dephases at the rate  $r$ , then  $N$  qubits in GHZ states will dephase at the rate  $Nr$ , which is why the connectivity of the GHZ state is  $N$ . By contrast, for RVB states, there are  $N/2$  singlets, so half of the qubits need to be removed to destroy entanglement. Similarly, this state is not markedly susceptible to dephasing, indicating a low value of connectivity.

Given that macroscopic entanglement exists, an important technological question is how easy this entanglement would be to extract and use. Suppose that two neutron beams are aimed at a magnetic substance, each at a different section<sup>31</sup>. It is fruitful — albeit not entirely mathematically precise — to think of this interaction as a state swap of the spins of the neutrons and the spins of the atoms in the solid. If the atoms in the solid are themselves entangled, then this entanglement is transferred to each of the scattered spins. This transferral could then presumably be used for further information processing. Similarly, schemes can be designed to extract entanglement from Bose–Einstein condensates<sup>32,33</sup> and superconductors<sup>34</sup> (which can be thought of as Bose–Einstein condensates of Cooper pairs of electrons), although none of these extraction schemes has been implemented as yet.

There are many open questions regarding entanglement. Here I have stated that, in theory, entanglement can exist in arbitrarily large and hot systems. But how true is this in practice? Another question is whether the entanglement of massless bodies fundamentally differs from that of massive ones<sup>35</sup>. Furthermore, does macroscopic entanglement also occur in living systems and, if so, is it used by these systems?

Some of the open questions might never be answered. Some might turn out to be uninteresting or irrelevant. One thing is certain though: current experimental progress is so rapid that future findings will surprise researchers and will take the current knowledge of entanglement to another level. ■

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